UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 5TH FEBRUARY 2004

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
   Candidates in Scotland must be in S4 or below.
   Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK IMC is about solving interesting problems, not about lucky guessing.

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1. What is the value of 4002 – 2004?

A 2004 B 2002 C 2000 D 1998 E 1996

2. You are told that 30 pupils have 25 different birthdays between them. What is the largest number of these pupils who could share the same birthday?

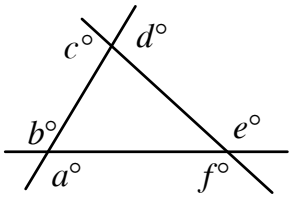
A 2 B 3 C 4 D 5 E 6

3. Four of these numbers can make two pairs so that each pair adds up to 98 765. Which number is the odd one out?

A 37 373 B 45 678 C 53 087 D 61 392 E 70 082

4. What is the value of $a + b + c + d + e + f$?

A 360 B 540 C 720 D 900
E it depends on the triangle



5. The sum of two numbers is 2. The difference between them is 4. What is their product?

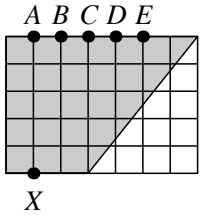
A −8 B −3 C 0 D 3 E 8

6. In Niatirb they use Cibara numerals. These are the same shape as normal Arabic numerals, but with the meanings in the opposite order. So "0" means "nine", "1" means "eight" and so on. But they write their numbers from left to right and use arithmetic symbols just as we do. So, for example, they use 62 for the number we write as 37.
How do the inhabitants of Niatirb write the answer to the sum which they write as 837 + 742?

A 419 B 580 C 1579 D 5317 E 8420

7. Which of the following straight lines cuts the shaded area in half?

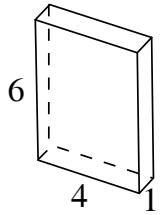
A *XA* B *XB* C *XC* D *XD* E *XE*

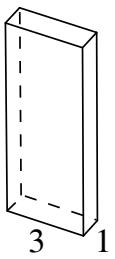


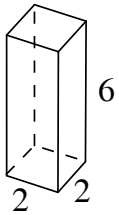
8. In March 2003 Welshman Tony Evans dropped a ball from an aircraft a mile above the Mojave desert to see if it would bounce. The ball was made from 6 million rubber bands, had a circumference of 14 ft 8 in, weighed 2600 pounds and took Mr Evans five years to build. On average, roughly how many rubber bands did he add each day whilst building the ball?

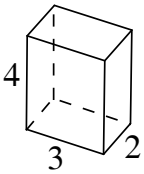
A 3 B 33 C 330 D 3300 E 33 000

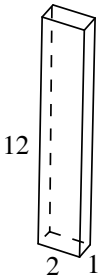
9. The cuboids below all have the same volume. Which of them has the greatest surface area?

A 

B 

C 

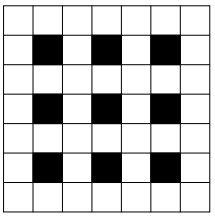
D 

E 

10. What is the mean of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$?

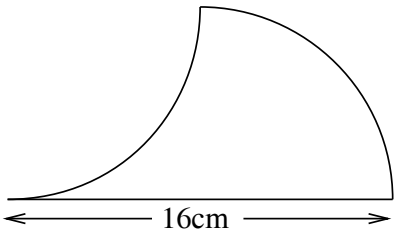
A $\frac{1}{5}$ B $\frac{1}{15}$ C $\frac{5}{12}$ D $\frac{7}{24}$ E $\frac{5}{16}$

11. The diagram shows a square board in which strips of white squares alternate with strips of black and white squares. A larger board, constructed in the same way, has 49 black squares. How many white squares are there on the larger board?



A 176 B 196 C 245 D 289 E 392

12. This figure is made from a straight line 16 cm long and two quarter circles, one with its centre at the midpoint of the straight line. What is the area of the figure (in cm^2) ?



A 64 B 16π C $32 + 16\pi$
D 32π E $16 + 8\pi$

13. Four of these points lie on a single straight line. Which is the odd one out?

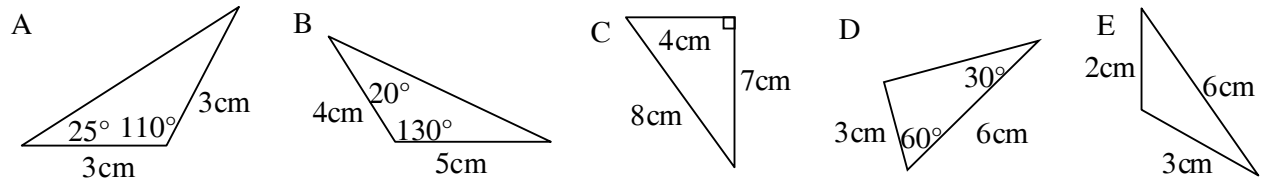
A (−3, −3) B (−2, −1) C (2, 5) D (4, 11) E (5, 13)

14. In this addition sum, each letter represents a different non-zero digit. What is the value of $a + w + a + y$?

$$\begin{array}{r} f\ l\ y \\ +\ f\ l\ y \\ +\ f\ l\ y \\ \hline a\ w\ a\ y \\ \hline \end{array}$$

A 13 B 15 C 16 D 17 E 18

15. Only one of these triangles can actually be made. Which is it?

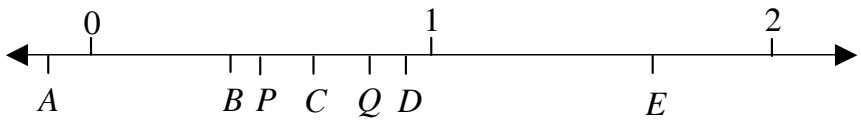


16. If the pattern shown is continued, what number will appear directly below 400?

A 438 B 439 C 440 D 441 E 442

			1			
		2	3	4		
	5	6	7	8	9	
10	11	12	13	14	15	16

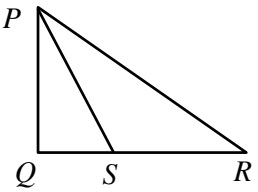
17.



A, *B*, *C*, *D*, *E*, *P* and *Q* are points on the number line as shown. One of the points represents the product of the numbers represented by *P* and *Q*. Which is it?

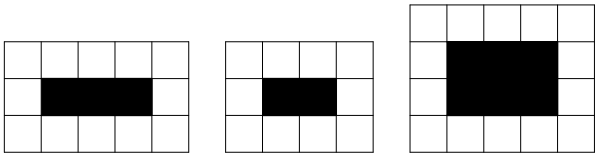
A B C D E

18. In the triangle PQR, there is a right angle at Q and angle QPR is 60°. The bisector of the angle QPR meets QR at S, as shown.
What is the ratio $QS : SR$?



A 1:1 B $1:\sqrt{2}$ C $1:(3-\sqrt{3})$ D $1:\sqrt{3}$ E 1:2

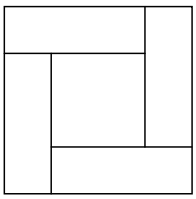
19. Three rectangular-shaped holes have been drilled passing all the way through a solid $3 \times 4 \times 5$ cuboid. The diagrams show the front, side and top views of the resulting block.
What fraction of the original cuboid remains?



A $\frac{13}{30}$ B $\frac{7}{15}$ C $\frac{1}{2}$ D $\frac{8}{15}$ E $\frac{17}{30}$

20. What is the largest power of 2 that divides $127^2 - 1$?
A 2^1 B 2^7 C 2^8 D 2^{63} E 2^{127}

21. A square is divided into four congruent rectangles and a smaller square, as shown. (The diagram is not to scale.) The area of the small square is $\frac{1}{4}$ of the area of the whole square. What is the ratio of the length of a short side of one of the rectangles to the length of a long side?

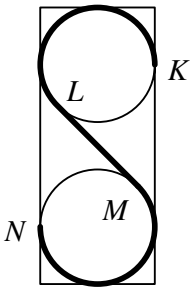


A $1:\sqrt{2}$ B $1:\sqrt{3}$ C 1:2 D 1:3 E 1:4

22. In a maths exam with N questions, you score m marks for a correct answer to each of the first q questions and $m + 2$ marks for a correct answer to each of the remaining questions. What is the maximum possible score?

A $(m+2)N - 2q$ B Nm C $mq + (m+2)q$ D $N(m+1)$ E $Nm + q(m+2)$

23. In the diagram, the letter S is made from two arcs, KL and MN, which are each five-eighths of the circumference of a circle of radius 1, and the line segment LM, which is tangent to both circles. At points K and N, common tangents to the two circles touch one of the circles. What is the length LM?

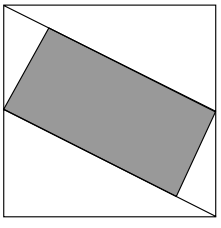


A $\frac{3}{2}$ B $3 - \sqrt{2}$ C 2
D $\frac{3\sqrt{2}}{2}$ E $1 + \sqrt{2}$

24. If p, q and $p - q$ are all positive integers, which of the following is least?

A $\frac{q^2}{p^2}$ B $\frac{p^2}{q^2}$ C $\frac{q}{p}$ D $\sqrt{\frac{q}{p}}$ E $\sqrt{\frac{p}{q}}$

25. The diagram shows a square with two lines from a corner to the middle of an opposite side. The rectangle fits exactly inside these two lines and the square itself. What fraction of the square is occupied by the shaded rectangle?



A $\frac{1}{3}$ B $\frac{2}{5}$ C $\frac{3}{10}$ D $\frac{1}{2}$ E $\frac{3}{8}$